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Computing based on Quantum Set Theory[1]

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Abstract: Computing in the framework of the quantum set theory of Gaisi Takeuti is discussed in this paper. We conclude, that computing based on quantum set theory can compute different alternatives in parallel, in this way offering the possibility of an increased computing capacity.

INTRODUCTION

Let L denote the lattice of all closed linear subspaces of a Hilbert space H. By John von Neumann [1] it is called quantum logic, the intrinsic logic of the quantum world. Gaisi Takeuti [2] showed that set theory based on von Neumann´s quantum logic (named quantum set theory) satisfies the generalization of the ZFC axioms (Zermelo, Frenkel plus Axiom of Choice) of set theory. Therefore a reasonable mathematics can be derived from quantum set theory but a much richer mathematics, a „gigantic" mathematics by the words of Takeuti.
In the paper [3] the basic mathematical problems of quantum field theory were collected and a way out of these difficulties, based on by repeating von Neumann´s line of thoughts for finitely many degrees of freedom, i. e. for quantum mechanics, was outlined.

The two main points of von Neumann are here:
a) He observed [4] that both of Schrödinger´s wave mechanics and Heisenberg´s matrix mechanics can be built up from so called elementary observables ("propositions") taking the values 0 and 1, since both the Schrödinger´s differential operators and the Heisenberg´s matrices are operators acting on Hilbert spaces and one can spectrally build up them from orthogonal projectors representing these elementary observables in the corresponding Hilbert space.
b) The canonical commutation relations have unique solutions up to unitarily equivalence, i. e. if one chooses either Schrödinger´s wave mechanics or Heisenberg´s matrix mechanics one gets the same results. The two representations are unitarily equivalent (**von Neumann´s theorem**). (This theorem guarantees, roughly speaking, the stability of quantum theory for finitely many degrees of freedom.)

For infinitely many degrees of freedom an alternative solution was proposed four decades ago.
A) The elementary propositional systems of local field theories were investigated and found that these propositions can not only take the values 0 and 1 but they have (infinitely many) third values, too, the so called **true-false** values. Thus, in the case of systems with infinitely many degrees of freedom, von Neumann´s line of thoughts steps beyond the mathematics based on the two valued logic [5].
B) The representations of the elementary propositional systems were looked for and the solutions of the commutation relations were studied on these representations. For this reason one had to turn to the extension of the basic tools of the theory of Hilbert spaces. Then it was found that the extended form of the von Neumann´s theorem holds true on these representations [6(b)]. (So in this way the quantization of local fields could lead, roughly speaking, to a stable quantum field theory [6].)

Note: A local field theory consists generally of an infinite collection of (identical) systems of finitely many degrees of freedom connected in space [6(b)].

This alternative solution of quantized fields with infinitely many degrees of freedom uses (based on) the „gigantic" mathematics derivable from the quantum set theory of Takeuti [2] [6].

It is a natural question that what is about computing in this framework? Let we discuss this question.

Computing in term of quantum set theory

1**. *Remarks about quantum computing*:**
1.1 The classical bit (0, 1) is an observable. One can derive all observables from them as elementary observables (considering quantum systems with finitely many degrees of freedom and arbitrary classical systems) [5]. However the quantum bit (0, superpositions, 1) is not an observable but a two dimensional state space, thus one could directly not derive all observables from them.
1.2 *Conjecture:* The quantum bits generate a state space with a Fock space structure, therefore, as we guess, this approach might knock against the Haag-theorem [3, 6], i. e. it might not be able to describe, in a mathematically rigorous way, interacting fields only free fields.
1.3 Then one could conclude from points 1.1 and 1.2 that the concept of the quantum bit may be incomplete. While the classical bit satisfies the completeness criterion, i. e. all observables can be derived from them (in the cases of arbitrary classical systems and quantum systems with finitely many degrees of freedom), the quantum bit does not satisfy the completeness criterion: one could not derive all observables from them in the cases of quantum systems with infinitely many degrees of freedom.

1 *In memory of John von Neumann for the 120th anniversary of his birthday.*

2**. Computing based on quantum set theory:**

2.1 One can build this approach on the quantum logic of von Neumann [1] and on the quantum set theory of Takeuti [2] instead of the notion of the quantum bit. This approach generalizes the real numbers. It turns from mathematics based on the two valued logic to mathematics based on the quantum logic, more precisely mathematics based on Takeuti´s quantum set theory. In references [5, 6, 7] it was shown that the propositional systems of quantum local field theories [consisting of an infinite collection of (identical) quantum systems of finitely many degrees of freedom connected in space] are the characteristic structures of this mathematics (let we call it „quantum mathematics").

2.2 Then in the framework of this "quantum mathematics" one should construct (of course, apart from the special case when H is two dimensional, it is yet an open problem waiting for a solution) the set up of a computer in parallel to the von Neumann´s set up of the computer.

2.3 In quantum set theory the truth values of the sentences are evaluated by the quantum logic. Then in this approach the task of the computer should be to evaluate, to compute the truth values of the statements. In this way the task of the computing should be the formulation of statements (programing) and the evaluation of them by the computer (running the program). The "quantum processor" should be built on the quantum logic (of the basic Hilbert space[2]). So in this von Neumann´s meaning the computer based on the two valued logic (simply named it classical computer) mechanizes the mathematics based on the two valued logic, while the computer based on the quantum logic (let we call it here "quantum computer"[3]) should (could?) mechanize the mathematics based on the quantum logic, the "quantum mathematics".

2.4 Since in the "quantum mathematics" the real numbers defined by Dedekind's cuts are self-adjoint operators of the basic Hilbert space H [2], thus the "quantum real numbers" are self-adjoint operators and the algebra of them is the algebra of these operators. The binary numbers are replaced by the "quantum binary numbers", namely in symbols $(0, 1) \rightarrow (0, e(x), 1)$ [$e^2(x) = e(x)$, the orthogonal projector of the closed linear subspace x of H, i.e. x is an element of L]. Therefore in symbols: the machine-made code of a classical program has the form of (1, 0, 0, 1, 1, ….) then the machine-made code of a "quantum program" should (could?) have the form of (e(x), 1, 0, e(y), e(z), …, 0, ...).

2.5 One can find the illustration of the geometrical structure of the system´s local state space both in references [6 (b), p. 198] and [7 (b), p. 1059]. Thus one might think of this structure as a „non commutative" Hilbert bundle. Then we arrive at the main result of this paper:

The local states of the system [*consisting of an infinite collection of (identical) quantum systems of finitely many degrees of freedom connected in space*] are sections of the bundle. The time evolution of these local states is governed, instead of the global/total Hamiltonian, by the local Hamiltonian of the system according to the eq. (5.8) in ref. [6 (b)] or eq. (30) in ref. [7 (b)]. This geometrical structure and time evaluation equations implies that:

2.6. Proposition: *D*ifferent alternatives [*for the individual members of the infinite collection of (identical) quantum systems of finitely many degrees of freedom connected in space*] given by an initial value of the evolution equation described by a section of the „non commutative" Hilbert bundle can be computed in *parallel*.

2 Again, in this approach the basic Hilbert space is not restricted to a two dimensional state space. Naturally, in the special case when H is two dimensional, the quantum processor is also built on the quantum logic of the basic Hilbert space H.

3 We use the name quantum computer though, as we see, it is more general in principle than the quantum computer based on quantum bits.

CONCLUSION

We can conclude, that a) computing based on quantum set theory offers a more general framework than the one based on the notion of the quantum bit, and as a corollary b) after solving the open problem of point 2.2 above, beyond the special case when H is two dimensional, it could and should offer a computing machinery exceeding the capacity of the computers we are using in this decades.

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